**Applications of Derivatives – Formula Sheet:**

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|  | **Average Rate of Change:**$$m\_{secant}=\frac{f\left(b\right)-f(a)}{b-a}$$**Tangent Line Equation:**$$y-y\_{1}=m(x-x\_{1})$$ |
|  | **Instantaneous Rate of Change:**$$m\_{tangent}=f^{'}\left(c\right)$$**The Normal Line:**$$m\_{normal}=-\frac{1}{m\_{tangent}}$$ |
|  | **Critical Points:**$$f^{'}\left(c\right)=0 or f^{'}\left(c\right)=DNE$$ |
|  | **Rolle’s Theorem:**$$1. f\left(x\right) is continuous on [a, b]$$$$2. f\left(x\right) is differentiable on (a, b)$$$$3. f\left(a\right)=f(b)$$If the 3 conditions above are met, then there is a number c in (a, b) where $f^{'}\left(c\right)=0.$ |
|  | **Mean Value Theorem:**If f(x) is continuous on [a, b] and differentiable on (a, b), then there is a number c in (a, b) such that$$f^{'}\left(c\right)= \frac{f\left(b\right)-f(a)}{b-a}$$$$m\_{tangent}= m\_{secant}$$ |

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|  | **Increasing/Decreasing Test:**$$1. f\left(x\right) is increasing when f^{'}\left(x\right)>0.$$$$2. f\left(x\right) is decreasing when f^{'}\left(x\right)<0.$$$$3. If f^{'}\left(x\right) does not change sign at c, then f\left(x\right) has no $$$$relative minimum or maximum at c.$$ |
|  | **First Derivative Test:**$$1. If f^{'} changes from+to-at c, then f has a local max. at c.$$$$2. If f^{'}changes from-to+at c, then f has a local min. at c.$$$$3. If f^{'}doesn't change sign at c, then f has no local min. or max. at c.$$ |
|  | **Concavity Test:**$$1. f is concave up when f^{''}(x)>0 or when f^{'}(x) is increasing.$$$$2. f is concave down when f^{''}(x)<0 or when f^{'}\left(x\right) is decreasing.$$ |
|  | **Inflection Points:**$$If f^{''}\left(c\right)=0 or f^{''}\left(c\right) does not exist, then there is an inflection$$$$point at c if the concavity changes at c.$$Note: $f must be continuous near c.$ |
|  | **Second Derivative Test:**$$1. If f^{'}\left(c\right)=0 and f^{''}\left(c\right)>0, then f has a relative minimum at c.$$$$2. If f^{'}\left(c\right)=0 and f^{''}\left(c\right)<0, then f has a relative maximum at c.$$Note: $f^{''} must be continuous near c.$ |
| **L’hospital’s Rule:**$$\lim\_{x \to a}\frac{f(x)}{g(x)}=\lim\_{x\to a}\frac{f'(x)}{g'(x)}$$Note: $g^{'}\left(x\right)\ne 0 near x=a.$ | **Newton’s Method for Approximating the Zeros of a Function:**$$x\_{n+1}= x\_{n}- \frac{f(x\_{n})}{f'(x\_{n})}$$ |

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| **Differentials:**$$dy=f^{'}\left(x\right) dx$$dx 🡪 Differential of xdy 🡪 Differential of y**Note:** $∆x=dx and ∆y≈dy$$$∆y=y\_{2}-y\_{1}=f\left(a+∆x\right)-f(a)$$ | **Tangent Line Approximations:** $Point (a, f\left(a\right))$$$y=f\left(a\right)+f'(a)(x-a)$$**Linear Equation – Point Slope Form:** $Point (x\_{1},y\_{1})$$$y-y\_{1}=m(x-x\_{1})$$**Note:** $ m=f^{'}\left(a\right) a=x\_{1} f\left(a\right)= y\_{1}$ |
| **Description:**C(x) 🡪 Cost Functionc(x) 🡪 Average Cost FunctionC’(x) 🡪 Marginal Cost Function**Note:** $c\left(x\right) is a minimum when C^{'}\left(x\right)=c(x)$ | **Average Cost Function:**$$c\left(x\right)=\frac{C(x)}{x}$$ |
| **Revenue Function:**$$R\left(x\right)=x∙p(x)$$**Note:** $p\left(x\right) is the price \left(demand\right)function.$ | **Profit Function:**$$P\left(x\right)=R\left(x\right)-C(x)$$**Note:** $Max profit occurs when R^{'}\left(x\right)=C'(x)$ |
| **Description:**R’(x) 🡪 Marginal RevenueP’(x) 🡪 Marginal ProfitC’(x) 🡪 Marginal Cost | **Marginal Profit:**$$P^{'}\left(x\right)=R^{'}\left(x\right)-C'(x)$$ |
| **Average Velocity:**$$\overbar{v}=\frac{s\left(b\right)-s(a)}{b-a}$$ | **Average Acceleration:**$$\overbar{a}=\frac{v\left(b\right)-v(a)}{b-a}$$ |
| **Instantaneous Velocity:**$$v\left(t\right)=s'(t)$$ | **Instantaneous Acceleration:**$$a\left(t\right)=v'(t)$$ |
| **Displacement:**$$d=s\left(b\right)-s(a)$$ | **The Position Function:**$$s(t)$$ |